AI-Driven Fuzzy Analogical Strategy for Pinch-Based Optimization of Heat Exchanger Networks in Aromatic Plants

Mostafa H. Hussein*[a] , Ghada Kadry[a], Heba A. El- Gawad[b]

ABSTRACT

This paper presents a novel AI-assisted approach to Heat Exchanger Network (HEN) synthesis using fuzzy analogical gates. The proposed methodology involves three key steps: (1) normalization of critical design variables, (2) evaluation using a fuzzy analogical gates network comprising symmetric (AND) and asymmetric (Invoke) gates, and (3) selection of the optimal minimum approach temperature based on a computed weight index. The symmetric gate integrates the hot utility requirements along with ΔTmin, while the second gate combines this output with cold utility demand. The method was validated using real aromatic plant case study. Results demonstrate that this technique reliably identifies optimal design parameters, reduces total annual cost, and is simple enough for manual implementation—offering a competitive alternative to more complex optimization models.

Keywords: Energy saving, HENs, Pinch Technology, Heat Integration, Fuzzy Analogical Gates, Decision Making.

1. INTRODUCTION

Heat Exchanger Network (HEN) synthesis is a critical aspect of process integration in chemical and petrochemical industries, aimed at minimizing energy consumption and optimizing utility usage. Since the introduction of the pinch analysis concept in the 1970s, numerous techniques have been developed to improve energy recovery, reduce operating costs, and enhance process efficiency [1-3].

Early approaches, such as pinch analysis [4-9] utilized thermodynamic principles and heuristic rules to identify energy-saving opportunities. These were followed by optimization-based methods, including linear programming (LP), nonlinear programming (NLP), mixed-integer linear programming (MILP), and mixed-integer nonlinear programming (MINLP). More recently, hybrid methods and heuristic algorithms have emerged to address the combinatorial complexity of large-scale HEN problems. However, a few of these approaches combine AI-inspired reasoning with interpretable, hand-calculable strategies

[a] Chemical Engineering Department, Higher Institute of Engineering, El-Shorouk Academy, Cairo, Egypt.

[b] Department of Engineering Mathematics and Physics, Higher Institute of Engineering, El Shorouk Academy, Cairo, Egypt.

*Corresponding author: Mostafa H. Hussein, Chemical Engineering Department, Higher Institute of Engineering, El-Shorouk Academy, Cairo, Egypt. ; E-mail: m.hassanein@sha.edu.eg

Despite extensive progress, many existing methods are either computationally intensive or require specialized optimization software. There remains a need for a reliable, accurate, and easy-to-implement approach, especially in preliminary design stages or in academic environments with limited computational tools [10-19].

This paper proposes a novel fuzzy logic-based strategy using analogical gates for determining the optimum minimum approach temperature in HEN design. The method leverages both symmetric (AND) and asymmetric (Invoke) gates to integrate key performance variables— ΔTmin, hot utility, and cold utility—into a decision framework. By applying fuzzy analogical reasoning, the method enables accurate prediction of optimal design conditions with minimal computational burden.

The proposed strategy is validated through two benchmark case studies commonly cited in the literature. Comparative analysis with previous works demonstrates the method's robustness, economic effectiveness, and simplicity of application.

2. ANALOGICAL GATES

Analogical gates are mathematical constructs inspired by logical gate behavior, designed to process continuous variables within a fuzzy logic framework. In this work, two types of analogical gates are used: **symmetric and asymmetric**.

2.1 Symmetric Gates (AND Gate)

The symmetric analogical gate models logical intersection behavior, where output increases only when both inputs increase simultaneously [20-21]. This is analogous to the behavior of a fuzzy AND gate. The mathematical formulation is:

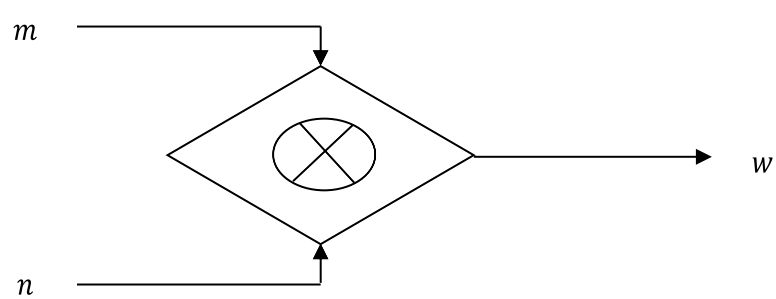

Figure .1(a) Symbols for the analogical (AND) gate

$$w = m \otimes n = m[1 - \xi(n,m)] + n[1 - \xi(m,n)] \quad (1)$$

$$\xi(n,m) = exp\left[\frac{an^2+bnm}{n^2+m^2}\right] \ and \ m,n \ \epsilon \ R \quad (2)$$

$$\xi(m,n) = exp\left[\frac{am^2+bmn}{n^2+m^2}\right] \ and \ m,n \ \epsilon \ R \quad (3)$$

In this formulation, x and y are normalized inputs (e.g., minimum approach temperature and minimum hot utility), and constants a and b control the gate sensitivity. In this study, empirically derived values of a = 2.28466 and b = - 0.089817 were used.

The key behavior of the symmetric gate:

- No output is produced if either input is zero.
- Output increases rapidly when both inputs are high.

2.2 Asymmetric Gates (Invoke Gate)

The asymmetric gate (also referred to as the Invoke gate) introduces priority among inputs. One input (typically the output from the AND gate) is given dominance, while the second input (e.g., cold utility) has a supporting influence.

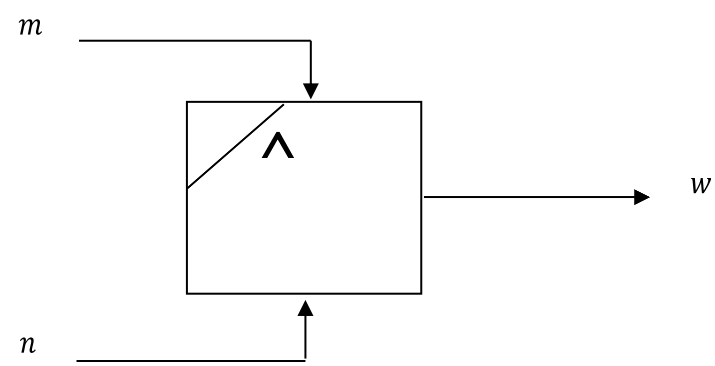

Fig.1 (b) Symbols for the analogical Invoke gate

The output of the Invoke gate is given by:

$$w = m \wedge n = m\xi_1[(n,m)] + n[1 - \xi_2(m,n)] \quad (4)$$

$$\xi_1(n,m) = exp\left[\frac{-(\alpha_1 n^2 + \beta_1 nm)}{n^2+m^2}\right] \; and \; m,n \in R \quad (5)$$

$$\xi_2(m,n) = exp\left[\frac{-(\alpha_2 n^2 + \beta_2 mn)}{m^2+n^2}\right] \; and \; m,n \in R \quad (6)$$

The values used for the constants are:

$(\alpha_1 = 1.4749267, \beta_1 = 0.92870491, \alpha_2 = 2.6317713, \beta_2 = 0.2287955)$

Key properties of the Invoke gate:

- Output is inhibited when the dominant input is absent.
- When the dominant input is present, it controls the output.
- The secondary input contributes when the dominant input is weak.

2.3 Gate Behavior and Implications

These fuzzy analogical gates enable soft decision-making by combining key variables in a nonlinear yet interpretable manner. The AND gate models have simultaneous importance, while the Invoke gate introduces a conditional hierarchy. Together, they provide robust computational logic for identifying the optimal approach temperature in heat exchanger network synthesis.

3. FUZZY ANALOGICAL GATES STRATEGY

The fuzzy analogical gates strategy provides a systematic approach for determining the optimum minimum approach temperature (ΔTmin) in Heat Exchanger Network (HEN) synthesis. The method combines normalized thermodynamic variables with fuzzy logic to evaluate different ΔTmin scenarios and select the most cost-effective one [32-34]. The strategy is divided into three sequential steps:

3.1 Step 1: Normalization of Input Variables

Three key variables influence the selection of ΔT_{min}:

- μ_1: Area
- μ_2: $Q_{H\ min}$
- μ_3: $Q_{C\ min}$

For each ΔTmin value, the corresponding QH_{min} and QC_{min} values are determined using a cascade diagram, as described in Linnhoff and Hindmarsh [7]. These values are then normalized using the following formula:

$$\mu = \frac{[f - f_{\min}]}{[f_{\max} - f_{\min}]}; \quad \text{Where, } \begin{Bmatrix} \mu = 0 \ \text{ if } \ f = f_{\min} \\ \mu = 1 \ \text{ if } \ f = f_{\max} \end{Bmatrix} \tag{7}$$

3.2 Step 2: Fuzzy Analogical Gates Network

The normalized variables are then passed through a two-layer fuzzy gate system:

- **First Gate (Symmetric / AND Gate)**: Combines μ_1 and μ_2 to evaluate simultaneous performance of ΔT_{min} and hot utility.
- **Second Gate (Asymmetric / Invoke Gate)**: Combines the output of the first gate with μ_3, emphasizing the influence of $QC_{\ min}$ in decision-making.

This sequential structure is illustrated in **Figure 2**, where:

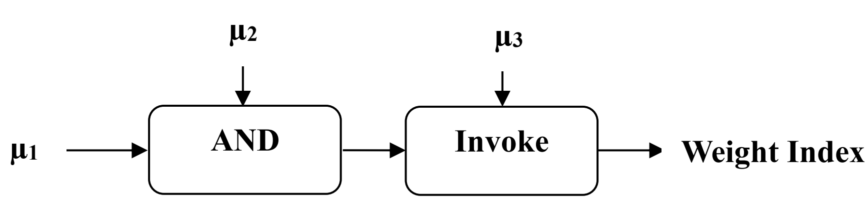

Figure 2. Flow diagram of the fuzzy analogical gates network used in HEN synthesis.

3.3 Step 3: Selection of the Best Weight Index

The final step is to identify the optimal ΔT_{min} by computing a **weight index (W.I.)** for each scenario and selecting the maximum value:

$$\text{Weight Index} = \max\{\omega_1, \omega_2, \omega_3 \ldots\ldots\} \quad (8)$$

The corresponding ΔT_{min} is considered optimal, as it balances energy utility demands with economic feasibility.

Summary

This approach integrates engineering insight with fuzzy decision-making tools, offering a flexible and efficient framework for HEN synthesis. By reducing reliance on complex mathematical solvers, the method can be executed manually or with simple computational tools.

4. PROBLEM STATEMENT

The objective of Heat Exchanger Network (HEN) synthesis is to design a network that enables effective heat recovery between process streams while minimizing energy consumption and overall cost. Given a set of hot streams (to be cooled) and cold streams (to be heated), along with associated thermodynamic and economic parameters, the goal is to determine:

- The optimal **(ΔT_{min})**
- The minimum **(Q_{Hmin})** and **(Q_{Cmin})** required
- A network configuration that minimizes the **(TAC)**, which includes utility and capital costs.

Design Inputs:

- Inlet and outlet temperatures of process streams
- Heat capacity flowrates (MCp) for each stream
- Overall heat transfer coefficients
- Cost parameters for utilities and equipment
- Economic factors such as plant life and interest rate

Traditional optimization approaches often involve complex nonlinear programming models or heuristic algorithms. In contrast, this work employs a **Fuzzy Analogical Gates Strategy** to determine the optimum ΔT_{min} based on the weighted contribution of normalized utility and temperature metrics.

By systematically analyzing various ΔT_{min} scenarios through the fuzzy gate system, the method identifies the one with the highest weight index, reflecting a balanced trade-off between energy recovery and cost. The selected ΔT_{min} is then used to construct a cost-effective HEN design.

5. CASE STUDY : AROMATICS PLANT PROBLEM

To evaluate the effectiveness of the proposed fuzzy analogical gates method, a benchmark problem from the literature was selected: the **Aromatics plant problem**. Thise case study are widely used in HEN research and provides a basis for comparison with other synthesis techniques such as MINLP, super-targeting, and stream-splitting strategies. This case study involves a more complex network with **four hot streams**, **five cold streams**, and utilities including **hot oil** and **water**.

Stream and plant data are listed in Tables 1 and 2. The cost Analysis of HEN at different ΔT_{min} for Aromatics HEN case is shown in Table 3.

Table 1. Stream specifications for **Aromatics HEN** case study (hot/cold streams and utilities).

Str	**Ts (°C)**	**T_t (°C)**	**M Cp (kW/°K)**	**h (kW/m²°K)**
H1	327	40	100	0.50
H2	220	160	160	0.25
H3	220	60	60	0.30
H4	160	45	400	0.18
C1	100	300	100	0.25
C2	35	164	70	0.27
C3	85	138	350	0.25
C4	60	170	60	0.15
C5	140	300	200	0.45
Hot Oil	330	250	-	0.30
Water	15	30	-	0.20

Table 2. Economic and design parameters used in the **Aromatics HEN** case

Utility data	Fuel gas cost	= 60 (US$/kW.yr)
	Cooling water cost	= 6 ($/kW.yr)
Plant Data	Rate of interest (i)	= 0 %
	Lifetime (n)	= 5 years
Capital cost data	Installed unit cost	= 10,000 + 350 A ($)

Table 3. Cost Analysis of HEN at different ΔT_{min} for Aromatics HEN case

ΔT	**Q_{Hmin}**	**Annual hot utility cost ($/Yr)**	**Q_{Cmin}**	**Annual cold utility cost ($/Yr)**	**Area (m²)**	**Capital cost ($)**	**Annualized total cost ($/yr)**
20	21680	1,300,800	29400	176,400	17514.8	1,506,304	2,983,504
22.5	23080	1,384,800	30800	184,800	15360.6	1,375,402	2,945,002
25	24480	1,468,800	32200	193,200	13881.5	1,291,018	2,953,018
30	27280	1,636,800	35000	210,000	11060.6	1,121,088	2,967,888
35	30080	1,804,800	37800	226,800	9049.9	1,042,377	3,273,977

the fuzzy method was applied to evaluate various ΔT_{min} values. As shown in Table 4, the optimal condition was found at **ΔT_{min} = 25°C**, corresponding to a weight index of **0.708**.

Table 4. Weight index calculations for different ΔT_{min} values using the fuzzy method **for Aromatics HEN case**

ΔT	**μ_1**	**μ_2**	**μ_3**	**ω**
20	0.000	1.000	1.000	0.000
22.5	0.333	0.833	0.833	0.286
25	0.667	0.667	0.667	**0.708**
30	0.833	0.333	0.333	0.302
35	1.000	0.000	0.000	0.000

The final design produced a utility requirement of **Q_H = 24.48 MW** and **Q_C = 32.20 MW**, with a **total area of 13,881 m²** and an **annualized total cost of $2.95 million/year**. The design is shown in **Figure 3**.

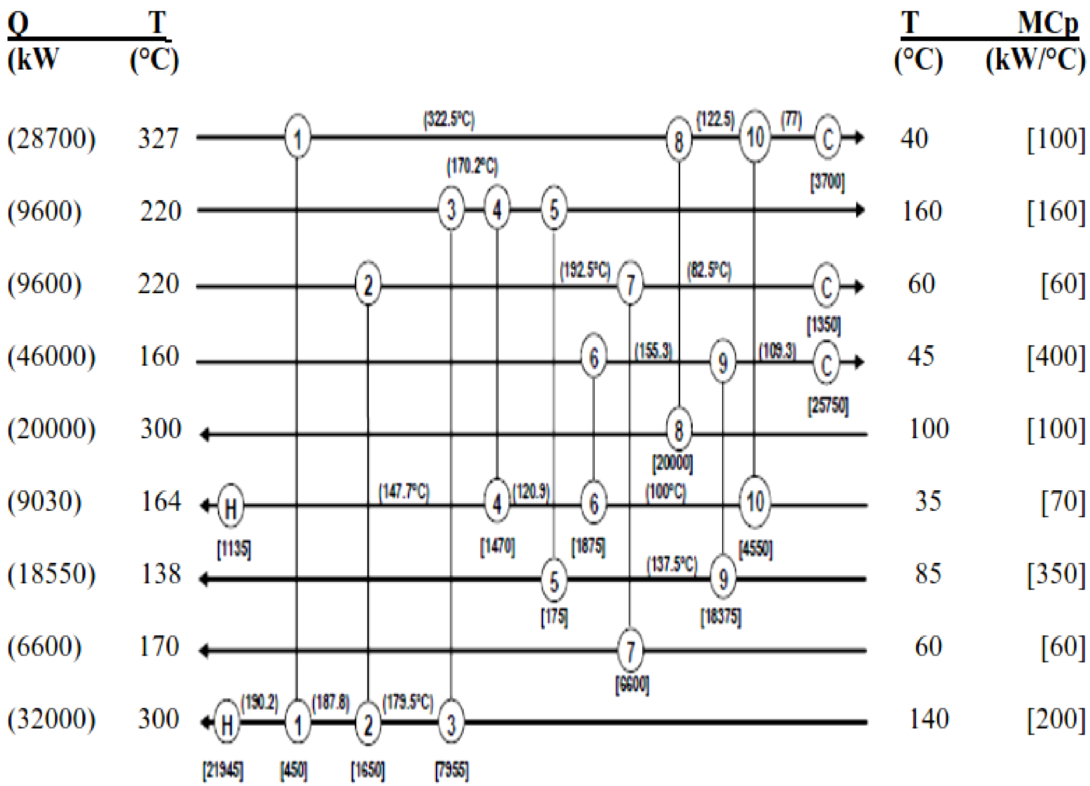

Figure.3. Grid diagram for the Aromatics HEN synthesis problem using the proposed fuzzy method at ΔT_{min} = 22.5°C, representing the final optimized configuration.

Table 5 compares this result to solutions from various authors. The fuzzy method outperformed all non-split solutions and was only slightly above the best stream-split design by Pettersson [24]. This confirms the method's robustness, especially given its reduced computational complexity.

Table 5. Annualized cost comparison for Aromatics HEN problem using different synthesis techniques.

Authors	Method	Stream Split	Area (m^2)	Q_{Hmin} (MW)	Q_{Cmin} (MW)	TAC (M$ /Yr)
Linnhoff [28]	Simple model	0	17,400	25.31	33.03	2.960
Zhu [29]	NLP	2	16,630	26.22	33.94	2.970
Zhu et al. [29]	NLP	0	16,380	26.83	34.55	2.980
Lewin [30]	GA	2	17,050	25.09	32.81	2.936
Lewin [31]	GA	0	16,880	25.69	33.41	2.946
Petterson [24]	SMR	7	17,473	24.27	31.99	2.905
Krishna [23]	DEM	0	16,536	25.88	33.60	2.942
Azeez [27]	STBS	7	—	—	—	2.922
Present work	**A.G**	**0**	**13,881**	**24.48**	**32.20**	**2.953**

Discussion

Across the real aromatic case study, the fuzzy analogical gates method demonstrated:

- Competitive total cost performance
- Simplicity in application (no stream splitting or advanced solvers required)
- Flexibility to evaluate multiple ΔT_{min} scenarios systematically

These results validate the method's usefulness in both academic and industrial HEN synthesis tasks, especially during early-stage conceptual design.

6. CONCLUSIONS

This work presents a lightweight, AI-inspired fuzzy logic framework for HEN synthesis using **fuzzy analogical gates**, a decision-making framework that integrates fuzzy logic with analogical reasoning. The method simplifies the process of selecting the optimal **minimum approach temperature (ΔT_{min})** by evaluating normalized values of key performance indicators—hot utility, cold utility, and targeted area through a two-layer gate system.

The proposed approach was applied to real Aromatics plant case study: the fuzzy method successfully identified ΔT_{min} values that resulted in **competitive total annual costs**, **without requiring stream splitting or complex solvers**. These findings validate the method's effectiveness, accuracy, and practicality. The method is particularly valuable for small-scale industries and energy audits where commercial optimization tools may not be available

Key advantages of the fuzzy analogical gates strategy include:

- **Simplicity**: Can be implemented manually or with basic programming tools.
- **Flexibility**: Applicable across a range of process scales and stream configurations.
- **Cost-effectiveness**: Produces low-cost solutions comparable to advanced optimization models.

While this method provides a strong foundation for preliminary design, future research could focus on:

- Extending the approach to **multi-period or dynamic HEN problems**
- Integrating it with **multi-objective optimization frameworks**
- Developing a **graphical software tool** to assist engineers in practical design settings

In summary, the fuzzy analogical gates method presents a reliable and accessible tool for energy integration, making it particularly valuable for early-stage design, academic instruction, or resource-constrained engineering environments.

NOMENCLATURE

AI	Artificial Intelligence
HENS	Heat Exchanger Networks
LP	Linear Programming
NLP	Non-Linear Programming
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Non-Linear Programming

W.I	Weight Index
EMAT	Exchanger Minimum Approach Temperature (°C)
HRAT	Heat Recovery Approach Temperature
IBMS	Interval Based MINLP superstructure
STBS	Supply and Target Based Superstructure
SMR	Sequential match reduction
SS	Stainless Steel
CS	Carbon Steel
A.G	Analogical Gates
GA	Genetic Algorithm.
DEM	Differential Evolution Method.
S.M	Simple Model
ΔT_{min}	Minimum Approach Temperature (°C)
Q_H	Minimum Hot Utility Requirement (kW)
Q_C	Minimum Cold Utility Requirement (kW)
TAC	Total Annualized Cost ($/year)
MCp	Heat Capacity Flow Rate (kW/°C)

μ_1, μ_2, μ_3 Normalized values of ΔT_{min}, Q_{Hmin}, and Q_{Cmin} respectively

7. REFERENCES

[1] Furman, K. C., and Sahinidis, N. V. "A critical review and annotated bibliography for heat exchanger network synthesis in the 20th century". Industrial and Engineering Chemistry Research, 41, (2335–2370), 2002. https://doi.org/10.1021/ie010389e

[2] Ravagnani, M. A. S. S., Silva, A. P., and Andrade, A. L. "Detailed equipment design in heat exchanger networks synthesis and optimization". Applied Thermal Analysis, Issue 23, (141–151), 2003. https://doi.org/10.1016/s1359-4311(02)00156-4

[3] Gundersen, T., and Grossmann, I. E. "Improved optimization strategies for automated heat exchanger network synthesis through physical insights". Comput. Chem. Eng., 14, (925-934), 1990. https://doi.org/10.1016/0098-1354(90)87050-y

[4] Linnhoff, B., and Flower, J. "Synthesis of heat exchanger networks. I, Systematic generation of energy optimal networks". AIChE Journal, 244, (633–642), 1978. https://doi.org/10.1002/aic.690240411

[5] Linnhoff, B., Mason, D. R., and Wardle, I. "Understanding heat exchanger networks". Comput. Chem. Eng. ,3, (295–302), 1979. https://doi.org/10.1016/0098-1354(79)80049-6

[6] Linnhoff, B., Towsend, D.W., Boland, D., Hewitt, G. F., Thomas, B. E. A., Guy, A. R., and Marsland, R. H. "A user guide on process integration for the efficient use of energy". UK: The Institute of Chemical Engineers. 1982. (I edition)

[7] Linnhoff, B., and Hindmarsh, E. "The pinch design method for heat exchanger networks". Chemical Engineering Science, 38, (745–763), 1983. https://doi.org/10.1016/0009-2509(83)80185-7

[8] Linnhoff, B. "Pinch analysis: A state-of-the-art overview". Transactions of the IChemE, 71Part A, (503–522), 1993.

[9] Linnhoff, B. "Use pinch analysis to knock down capital costs and emissions". Chemical Engineering Progress, (32–57). 1994.

[10] Grossmann, I. E., Caballero, J. A., and Yeomans, H. "Advances in mathematical programming for the synthesis of process systems". Latin American Applied Research, 30,(263–284), 2000.

[11] Cerda, J., and Westerberg, A. W. "Synthesizing heat exchanger networks having restricted stream/stream matches using transportation problem formulation". Chemical Engineering Science, 38, (1723-1730), 1983. https://doi.org/10.1016/0009-2509(83)85029-5

[12] Colberg, R. D., and Morari, M. "Area and capital cost targets for heat exchanger network synthesis with constrained matches and unequal heat transfer coefficients". Comput. Chem. Eng.14, (17-23), 1990. https://doi.org/10.1016/0098-1354(90)87002-7

[13] Floudas, C. A., Ciric, A. R., and Grossmann, I. E. "Automatic synthesis of optimum heat exchanger network configurations". AIChE Journal, 32, (276-286), 1986. https://doi.org/10.1002/aic.690320215

[14] Papoulias, S. A., and Grossmann, I. E. "structural optimizations approach in process synthesis. Part II. Heat recovery networks". Comput. Chem. Eng., 7, (707-716), 1983.

[15] Bjork, K., and Westerlund, T. "Global optimization for heat exchanger network synthesis problems with and without the isothermal mixing assumption". Comput. Chem. Eng., 26, (1581–1593), 2002. https://doi.org/10.1016/s0098-1354(02)00129-1

[16] Ciric, A. R., and Floudas, C. A. "Heat exchanger network synthesis without decomposition". Comput. Chem. Eng., 15, 385–396. 1991. https://doi.org/10.1016/0098-1354(91)87017-4

[17] Quesada, I. E., and Grossmann, I. E. "Global optimization algorithm for heat exchanger networks". Industrial Engineering Chemistry Research, 32, 487. 1993. https://doi.org/10.1021/ie00015a012

[18] Yee, T. F., and Grossmann, I. E. "Simultaneous optimization models for heat integration. II. Heat exchanger network synthesis". Comput. Chem. Eng., 14, 1165. 1990. https://doi.org/10.1016/0098-1354(90)85010-8

[19] Zamora, J., and Grossmann, I. E. "A global optimization algorithm for the synthesis of heat exchanger networks with no stream splits". Comput. Chem. Eng., 22, 367–384. 1998. https://doi.org/10.1016/s0098-1354(96)00346-8

[20] Badreddin E., "Fuzzy relations for behavior-fusion of mobile robots"., *Proc. IEEE conf.* on Robotics and Automation, San Diego, California,(1994), pp. 8-13. https://doi.org/10.1109/robot.1994.351066

[21] Aly S., "Fuzzy analogical gates for separation sequence synthesis". *Chem. Eng. Proc, 36* (1997) 209-217. https://doi.org/10.1016/s0255-2701(96)04191-8

[22] Shenoy, U.V., "Heat Exchanger Network Synthesis‟ Process Optimization by Energy and Resource Analysis. Gulf Publishing Company Houston, Texas.

[23] Krishna M. Yerramsetty , C.V.S. Murty , Synthesis of cost-optimal heat exchanger networks using differential evolution, Computers and Chemical Engineering 32 (2008) 1861–1876. https://doi.org/10.1016/j.compchemeng.2007.10.005

[24] Pettersson, F. (2005). Synthesis of large-scale heat exchanger networks using a sequential match reduction approach. *Computers & Chemical Engineering*, *29*(5), 993–1007. https://doi.org/10.1016/j.compchemeng.2004.11.001

[25] Ravagnani, M. A. S. S., Silva, A. P., Arroyo, P. A., & Constantino, A. A. (2005). Heat exchanger network synthesis and optimization using Genetic Algorithm. *Applied Thermal Engineering*, *25*, 1003–1017. https://doi.org/10.1016/j.applthermaleng.2004.06.024

[26] Hojjati, Mahmoud Reza and Omidkhah, Mohammad Reza,.‟‟ Cost Effective Heat Exchanger Network Design with Mixed Materials of Construction‟‟ Iran.J.Chem.& Chem.Eng. vol.23,No.2,2004.

[27] Azeez O.S , Isafiade A.J.,.Fraser D.M., „‟Supply and Target based superstructure synthesis of heat and mass exchanger networks‟‟.*Chemical Engineering Research & Design Part A*, *90* (2012), 266 – 287 https://doi.org/10.1016/j.cherd.2011.07.004

[28] Linnhoff, B., & Ahmad, S. (1990). Cost optimum heat exchanger networks. 1. Minimum energy and capital using simple models for capital cost. *Computers & Chemical Engineering*, *14*(7), 729–750 https://doi.org/10.1016/0098-1354(90)87083-2

[29] Zhu, X. X., O‟Neill, B. K., Roach, J. R., & Wood, R. M. (1995). A method for automated heat exchanger synthesis using block decomposition and non-linear optimization. *Chemical Engineering Research & Design Part A*, *73*(11), 919–930.

[30] Lewin, D. R. (1998). A Generalized method for HEN synthesis using stochastic optimization. II. The synthesis of cost-optimal networks. *Computers & Chemical Engineering*, *22*(10), 1387–1405. https://doi.org/10.1016/s0098-1354(98)00221-x

[31] Lewin, D. R., Wang, H., & Shalev, O. (1998). A Generalized method for HEN synthesis using stochastic optimization. I. General framework and MER optimal synthesis. *Computers & Chemical Engineering*, *22*(10), 1503–1513. https://doi.org/10.1016/s0098-1354(98)00220-8

[32] M. H. Hussein, H. Moselhy, S. Aly, and M. E. Awad, "Fuzzy Analogical Gates Approach for Heat Exchangers Networks," *Int. J. Comput. Appl.*, vol. 73, no. 21, 2013. https://doi.org/10.5120/13015-9646

[33] A.H. Ibrahim, M.E. Awad, S.Aly, M. H. Hussein, " Synthesis and Design of Natural Gas Transmission Networks through Fuzzy Analogical Gates," *Petroleum & Coal,* vol. 66, no. 2, 2024.

[34] M. A. Atef, S. M. Salem, and M. H. Hussein, "Enhanced Cascade Refrigeration System Performance via Fuzzy-Based Multi-Objective Optimization'. *Petroleum & Coal*, vol. 67, no.1, 2025